Building Tight Occurrence Nets from Reveals Relations

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Building Tight Occurrence Nets from Reveals Relations

1. Introduction and Preliminaries on Occurrence Nets
2. Reveals Relation and Facets Abstraction
3. ERL: A Logic for Occurrence Nets
4. From Occurrence Nets to ERL Formulas
5. From ERL Formulas to Occurrence Nets: A Synthesis Procedure
6. Conclusion
Building Tight Occurrence Nets from Reveals Relations

1. Introduction and Preliminaries on Occurrence Nets
   - Petri Nets and Occurrence Nets
   - Maximal Runs

2. Reveals Relation and Facets Abstraction

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4. From Occurrence Nets to ERL Formulas

5. From ERL Formulas to Occurrence Nets: A Synthesis Procedure

6. Conclusion
Introduction

- Occurrence nets model the concurrent behavior of Petri nets,
- Structural relations on events: causality, conflict and concurrency.

[Nielsen, Plotkin, Winskel, 1980]
Introduction

- Occurrence nets model the concurrent behavior of Petri nets,
- Structural relations on events: causality, conflict and concurrency.

[Nielsen, Plotkin, Winskel, 1980]

The structural relations imply logical dependencies between event occurrences:

- $a$ is a causal predecessor of $b \Rightarrow$ if $b$ occurs, then $a$ has occurred,
- $a$ and $b$ are in conflict $\Rightarrow$ if $b$ occurs, then $a$ does not occur,

But some logical dependencies cannot be expressed by the structural relations.
Introduction

- **Occurrence nets** model the concurrent behavior of Petri nets,
- **Structural relations on events**: causality, conflict and concurrency.

[Nielsen, Plotkin, Winskel, 1980]

The structural relations imply **logical dependencies** between event occurrences:

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- \( a \) and \( b \) are in conflict \( \Rightarrow \) if \( b \) occurs, then \( a \) does not occur,

But some logical dependencies cannot be expressed by the structural relations.

Objectives

- Formalization of these logical dependencies (extension of [Haar, 2010]),
- Synthesis problem: Build a net whose set of runs is described by a logical formula on events.
Petri Nets

Definition (Petri net)

A Petri net (PN) is a tuple \((P, T, F, M_0)\) where

- \(P\) and \(T\) are finite sets of places and transitions,
- \(F \subseteq (P \times T) \cup (T \times P)\) is the set of directed arcs and
- \(M_0 \subseteq P\) is the initial marking.

- pre-set of \(t \in T\): \(\bullet t \overset{\text{def}}{=} \{ p \in P \mid (p, t) \in F \}\)
- post-set of \(t \in T\): \(t^* \overset{\text{def}}{=} \{ p \in P \mid (t, p) \in F \}\)

\[M_0 = \{1, 4\}\]

\(\bullet a \subseteq M_0, \bullet b \subseteq M_0\)

\(a\) and \(b\) are enabled
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\(a\) and \(b\) are enabled
Processes, Branching Processes and Unfoldings

Process: representation of a non-sequential run as a partial order.
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Process: representation of a non-sequential run as a partial order.

causality: $a_1 \leq b_2$

concurrency: $(c_1 \not\leq d_1) \land (d_1 \not\leq c_1)$

$\Rightarrow c_1 \text{ co } d_1$
Processes, Branching Processes and Unfoldings

**Process:** representation of a non-sequential run as a partial order.

**Branching process:** representation of several runs.

**Unfolding:** maximal branching process.

**causality:** $a_1 \leq b_2$

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Processes, Branching Processes and Unfoldings

Process: representation of a non-sequential run as a partial order.

Branching process: representation of several runs.

Unfolding: maximal branching process.

causality: $a_1 \leq b_2$

concurrency: $(c_1 \not\leq d_1) \land (d_1 \not\leq c_1) \land \neg (c_1 \# d_1) \Rightarrow c_1 \co d_1$

conflict: $a_1 \# b_1, c_1' \# c_2'$
Three Structural Relations

The structure of an unfolding induces three relations over its events:

Causality $\leq$

- $e \leq f \iff e F^* f$
- $e \leq f \Rightarrow \text{any run that contains } f \text{ also contains } e$
- $e \not\leq f$
Three Structural Relations

The structure of an unfolding induces three relations over its events:

**Causality ≤**

\[ e \leq f \quad \overset{\text{def}}{\Leftrightarrow} \quad e \overset{F^*}{\leftrightarrow} f \]

\[ \Rightarrow \quad \text{any run that contains } f \text{ also contains } e \]


**Conflict #**

\[ e \#_d g \quad \overset{\text{def}}{\Leftrightarrow} \quad e \neq g \land \cdot e \cap \cdot g \neq \emptyset \]

\[ f \# h \quad \overset{\text{def}}{\Leftrightarrow} \quad \exists e \leq f, g \leq h : e \#_d g \]

\[ \Leftrightarrow \quad \text{no run contains both } f \text{ and } h \]
Three Structural Relations

The structure of an unfolding induces three relations over its events:

### Causality $\leq$

- $e \leq f \quad \overset{\text{def}}{\iff} \quad e \ F^* \ f$
- $\Rightarrow$ any run that contains $f$ also contains $e$
- $\nleq$

### Conflict #

- $e \ #_d \ g \overset{\text{def}}{\iff} \quad e \neq g \land \bullet e \cap \bullet g \neq \emptyset$
- $f \ # \ h \overset{\text{def}}{\iff} \exists e \leq f, g \leq h : e \ #_d \ g$
- $\Leftrightarrow$ no run contains both $f$ and $h$

### Concurrency $co$

- $f \ co \ i \overset{\text{def}}{\iff} \neg(i \ # f) \land \neg(i \leq f) \land \neg(f \leq i)$
Occurrence Nets [Nielsen, Plotkin, Winskel, 1980]

The structure of an unfolding is an occurrence net.

**Notations**

- **causal past of** $e$: $\lceil e \rceil \overset{\text{def}}{=} \{ f \in E \mid f \leq e \}$
- **conflict set of** $e$: $\#[e] \overset{\text{def}}{=} \{ f \in E \mid f \# e \}$

**Definition (Occurrence net)**

An occurrence net (ON) is a net $(B, E, F)$ where $B$ and $E$ are the sets of conditions and events, and such that:

1. $\forall e \in E, \neg (e \# e)$ (no self-conflict),
2. $\forall e \in E, \neg (e < e)$ (acyclicity),
3. $\forall e \in E, |\lceil e \rceil| < \infty$ (finite causal past),
4. $\forall b \in B, |\bullet b| = 1$ (no backward branching),
5. $\bot \in E$ is the only $\leq$-minimal node (event $\bot$ creates the initial conditions).
**Runs**

**Definitions (Run, Maximal run)**

A run (or configuration) $\omega$ of an ON is a set of events which is

- **conflict free**: $\forall e \in \omega, \#[e] \cap \omega = \emptyset$,

- **causally closed**: $\forall e \in \omega, [e] \subseteq \omega$.

A run is **maximal** iff it is maximal w.r.t. $\subseteq$.

$\Omega$ denotes the set of **maximal runs**.
Run

Definitions (Run, Maximal run)

A run (or configuration) $\omega$ of an ON is a set of events which is

- conflict free: $\forall e \in \omega, \#[e] \cap \omega = \emptyset$,
- causally closed: $\forall e \in \omega, \lceil e \rceil \subseteq \omega$.

A run is maximal iff it is maximal w.r.t. $\subseteq$.

$\Omega$ denotes the set of maximal runs.

Lemma: The conflict relation defines the maximal runs.

A set of events $\omega$ is a maximal run iff

$$\forall a \in E, a \notin \omega \iff \#[a] \cap \omega \neq \emptyset$$

i.e., a set of events which is conflict-free and maximal w.r.t. $\subseteq$ is a maximal run (the causal closure is implied).
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   - Tight Nets

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Reveals Relation \[\text{[Haar, 2010]}\]

**Definition (Reveals relation $\triangleright$)**

Event $e$ **reveals** event $f$, written $e \triangleright f$, iff $\forall \omega \in \Omega, (e \in \omega \Rightarrow f \in \omega)$.

**Causal closure**

$\forall e, f \in E, f \leq e \Rightarrow e \triangleright f$

**Progress assumption**

$\forall e, f \in E, e \triangleright f \iff \#[f] \subseteq \#[e]$

i.e. any event that could prevent the occurrence of $f$ is prevented by the occurrence of $e$.

$d \triangleright a$, $k \triangleright b$, $h \triangleright \bot$, $a \triangleright c$

For any run $\omega$, $a \in \omega \Rightarrow b \notin \omega$

$\Rightarrow \#[c] \cap \omega = \emptyset$

$\Rightarrow c \in \omega$
Reveals Relation [Haar, 2010]

Definition (Reveals relation $\triangleright$)

Event $e$ reveals event $f$, written $e \triangleright f$, iff $\forall \omega \in \Omega, (e \in \omega \Rightarrow f \in \omega)$.

Properties

- $\triangleright$ is reflexive and transitive, but it is not antisymmetric in general.
- The conflict relation ($\#$) is inherited under $\triangleright^{-1}$: $g \triangleright a \land a \# b \Rightarrow g \# b$. 

Facets Abstraction [Haar, 2010]

Definition (Facets)

A facet of an ON is an equivalence class of $\sim = \triangleright \cap \triangleright^{-1}$. 
Facets Abstraction [Haar, 2010]

**Definition (Facets)**
A facet of an ON is an equivalence class of $\sim = \triangleright \cap \triangleright^{-1}$.

**Definition (Reduced ON)**
A reduced ON is an ON $(B, \Psi, F)$ such that $\forall \psi_1, \psi_2 \in \Psi, \psi_1 \sim \psi_2 \iff \psi_1 = \psi_2$.

Facets can be contracted into events.
Binary Relations on $\Psi$

The causality ($\leq$), conflict ($\#$), concurrency ($co$) and reveals ($\triangleright$) relations naturally extend to $\Psi$.

Lemma 1

$\triangleright$ is a partial order on $\Psi$ ($\triangleright$ is antisymmetric by definition of a reduced ON).

$(\Psi, \triangleright^{-1}, \#)$ is not always an event structure

- $\triangleright^{-1}$ is a partial order, $\checkmark$
- $\forall \psi \in \Psi$, the set $\{\psi' \mid \psi \triangleright \psi'\}$ is finite,
- $\#$ is inherited under $\triangleright^{-1}$. $\checkmark$
Binary Relations on $\Psi$

The causality ($\leq$), conflict ($\#$), concurrency ($co$) and reveals ($\triangleright$) relations naturally extend to $\Psi$.

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$(\Psi, \triangleright^{-1}, \#)$ is not always an event structure

- $\triangleright^{-1}$ is a partial order, ✓
- $\forall \psi \in \Psi$, the set $\{\psi' \mid \psi \triangleright \psi'\}$ is finite, ✗
- $\#$ is inherited under $\triangleright^{-1}$. ✓
Infinite Revealed Set

For a facet $\psi$, the set $\{\psi' \mid \psi \triangleright \psi'\}$ may not be finite.

Facet $\psi_3$, associated with transition $t_3$, reveals all the facets $\psi_{1,i}$, $i \in \mathbb{N}^*$, associated with transition $t_1$. 
Binary Relations on $\Psi$

The causality $(\leq)$, conflict $(\#)$, concurrency $(co)$ and reveals $(\triangleright)$ relations naturally extend on $\Psi$.

**Lemma 1**

$\triangleright$ is a partial order on $\Psi$ ($\triangleright$ is antisymmetric by definition of a reduced ON).

**Lemma 2**

For any finite reduced ON $(B, \Psi, F)$, $(\Psi, \triangleright^{-1}, \#)$ is an event structure since:

- $\triangleright^{-1}$ is a partial order,
- $\forall \psi \in \Psi$, the set $\{ \psi' | \psi \triangleright \psi' \}$ is finite,
- $\#$ is inherited under $\triangleright^{-1}$.

**Lemma 3**

If $(B, \Psi, F)$ is the reduced unfolding of a safe Petri net, then $\triangleright^{-1}$ is well-founded on $\Psi$: there is no infinite chain of distinct facets $\psi_1 \triangleright \psi_2 \triangleright \psi_3 \ldots$.
Concurrency vs Logical Independency

Structural relations and logical dependencies

- \( a \not\equiv b \iff \text{for any run } \omega, \{a, b\} \not\subseteq \omega. \)
- \( a \leq b \implies \text{for any run } \omega, b \in \omega \implies a \in \omega (b \triangleright a), \)
- Does \( a \ co b \) mean \( a \) and \( b \) are logically independent?
  - No, they can be related by \( \triangleright \).

\[ c \ co a \text{ and } c \triangleright a \]
\[ a \ co b \text{ and } a \ ind b. \]
Concurrency vs Logical Independency

### Structural relations and logical dependencies

- $a \not\sim b \iff$ for any run $\omega$, $\{a, b\} \not\subseteq \omega$.
- $a \preceq b \Rightarrow$ for any run $\omega$, $b \in \omega \Rightarrow a \in \omega$ $(b \triangleright a)$,
- Does $a \text{ co } b$ mean $a$ and $b$ are logically independent?
  - **No**, they can be related by $\triangleright$.

### Independency relation $\text{ind}$

$$\forall a, b \in \Psi, \ a \text{ ind } b \overset{\text{def}}{\iff} \neg (a \not\sim b) \land \neg (b \triangleright a) \land \neg (a \triangleright b)$$

$$\iff a \text{ co } b \land \neg (b \triangleright a) \land \neg (a \triangleright b)$$
**Tight Nets**

**Definition (Tight net)**

A **tight net** is a reduced ON \((B, \Psi, F)\) such that \(\forall a, b \in \Psi, \; a \triangleright b \iff b \leq a\).

![Diagram of a tight net](image)

A reduced ON which is not tight because \(c \triangleright a\) and \(\neg (a \leq c)\).

![Diagram of a non-tight net](image)

A tight net.

In a tight net, \(\triangleright^{-1}\) is equivalent to \(\leq\), and \(\text{ind}\) is equivalent to \(\text{co}\).
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   - Syntax and Semantics
   - Extended Reveals Relation
   - Minimal Constraints

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6. Conclusion
A Logical Framework

- Relations between more than 2 events: “a and b reveal c”
- Logical formalization of these relations, leaving the structure aside.
ERL: Event Reveal Logic

Syntax

variables: $\Psi$ (including $\psi_\bot$, the facet of event $\bot$),

constants: $\{\text{tt}, \text{ff}\}$,

logical connectives: $\lor$, $\land$, $\rightarrow$, $\leftrightarrow$ and $\neg$,

well-formed formulas: $\varphi ::= \text{tt} | \text{ff} | \psi \forall \psi \in \Psi$

$\mid \neg \varphi \mid \varphi \land \varphi \mid \varphi \lor \varphi \mid \varphi \rightarrow \varphi \mid \varphi \leftrightarrow \varphi$

Semantics

For any set of facets $\omega \subseteq \Psi$,

- $\forall \psi \in \Psi$, $\omega \models \psi \overset{\text{def}}{=} \psi \in \omega$,

- $\neg$, $\lor$, $\land$, $\rightarrow$ and $\leftrightarrow$ have the usual semantics, in particular:

  $\omega \models \neg \varphi \overset{\text{def}}{=} \omega \not\models \varphi$, and $\omega \models \varphi_1 \rightarrow \varphi_2 \overset{\text{def}}{=} \omega \models \varphi_1 \rightarrow \omega \models \varphi_2$.

For any set of sets of facets $\Omega \subseteq 2^\Psi$,

- for any ERL formula $\varphi$, $\Omega \models \varphi \overset{\text{def}}{=} \forall \omega \in \Omega, \omega \models \varphi$
Extended Reveals Relation

Any well-formed formula can be brought into a conjunctive normal form:

\[
\bigwedge_{i \in I} \left( b_{1}^{i} \lor b_{2}^{i} \lor \cdots \lor b_{n_{i}}^{i} \lor \neg a_{1}^{i} \lor \neg a_{2}^{i} \lor \cdots \lor \neg a_{m_{i}}^{i} \right)
\]

iff

\[
\bigwedge_{i \in I} \left( \left( a_{1}^{i} \land a_{2}^{i} \land \cdots \land a_{m_{i}}^{i} \right) \rightarrow \left( b_{1}^{i} \lor b_{2}^{i} \lor \cdots \lor b_{n_{i}}^{i} \right) \right)
\]

iff

\[
\bigwedge_{i \in I} \left( \bigwedge_{a \in A} a \rightarrow \bigvee_{b \in B} b \right), \text{ where } A_{i} = \{a_{1}^{i}, \ldots, a_{m_{i}}^{i}\} \text{ and } B_{i} = \{b_{1}^{i}, \ldots, b_{n_{i}}^{i}\}.
\]

We can focus on formulas of the form \( \bigwedge_{a \in A} a \rightarrow \bigvee_{b \in B} b \).

\[
\Omega \models \bigwedge_{a \in A} a \rightarrow \bigvee_{b \in B} b \iff \forall \omega \in \Omega, A \subseteq \omega \Rightarrow B \cap \omega \neq \emptyset
\]

Definition (Extended reveals relation)

Let \( \Omega \subseteq 2^{\Psi} \) be a set of runs, and \( A, B \) two sets of facets. \( A \) reveals \( B \), written \( A \rightarrow B \), iff \( \forall \omega \in \Omega, A \subseteq \omega \Rightarrow B \cap \omega \neq \emptyset \).
Extended Reveals Relation

Definition (Extended reveals relation)

Let $\Omega \subseteq 2^\Psi$ be a set of runs, and $A, B$ two sets of facets. $A$ reveals $B$, written $A \rightarrow B$, iff $\forall \omega \in \Omega, A \subseteq \omega \Rightarrow B \cap \omega \neq \emptyset$.

Properties

- $\{a\} \rightarrow \{b\} \iff a \triangleright b$
- Conflicts can be expressed with this extended reveals relation: $\{a, b\} \rightarrow \emptyset \iff a \nmid b$. 
Extended Reveals Relation

Examples

\[ A \rightarrow B \iff \forall \omega \in \Omega, A \subseteq \omega \Rightarrow B \cap \omega \neq \emptyset \]
\[ \iff \Omega \models \bigwedge_{a \in A} a \rightarrow \bigvee_{b \in B} b \]

\{c, e\} \rightarrow \{a\}
\{c, d, e\} \rightarrow \{a\}
\{e', e\} \rightarrow \emptyset

\{a, b\} \rightarrow \{c', c, d\}
\{a\} \rightarrow \{c, d\}
\emptyset \rightarrow \{a, a'\}
Binary Minimal Constraints

### Immediate conflict relation $\#_i$

\[
 a \#_i b \iff (c \neq a \land a \triangleright c \land c \# b) \lor (c \neq b \land b \triangleright c \land c \# a)
\]

### Immediate binary reveals relation $\triangleright_i$

\[
 a \triangleright_i b \iff (c \neq a \land c \neq b \land a \triangleright c \land c \triangleright b)
\]

\(\neg (c \#_i a')\) because \(c \triangleright a\) and \(a \# a'\)

\(c \triangleright_i \psi_{\bot}\) because \(c \triangleright a\) and \(a \triangleright \psi_{\bot}\)

**Remarks**

- \(\triangleright = \triangleright^*_i\),
- \(\# = (\triangleright_i^{-1})^* \circ \#_i \circ \triangleright^*_i\) (\(\triangleright\)-inheritance of \(\#\)),
- Therefore \(\triangleright_i\) and \(\#_i\) define \(\Omega\).
A Synthesis Problem for Occurrence Nets

Notation

\[
[\varphi] \overset{\text{def}}{=} \{ \omega \subseteq \Psi \mid \omega \models \varphi \}
\]

Given a reduced ON \( \mathcal{N} \), build an ERL formula \( \Phi_{\mathcal{N}} \) such that \([\Phi_{\mathcal{N}}] = \Omega_{\mathcal{N}}\).
A Synthesis Problem for Occurrence Nets

Notation

$$[\varphi] \overset{\text{def}}{=} \{\omega \subseteq \Psi \mid \omega \models \varphi\}$$

1. Given a reduced ON $N$, build an ERL formula $\Phi_N$ such that $\llbracket \Phi_N \rrbracket = \Omega_N$,

2. Given an ERL formula $\varphi$, build a reduced ON $N'$ such that $\Omega_{N'} \equiv \llbracket \varphi \rrbracket$ (if such a reduced ON exists),
A Synthesis Problem for Occurrence Nets

Notation

\[ [\varphi] \overset{\text{def}}{=} \{ \omega \subseteq \Psi \mid \omega \models \varphi \} \]

1. Given a reduced ON \( \mathcal{N} \), build an ERL formula \( \Phi_\mathcal{N} \) such that \( [\Phi_\mathcal{N}] = \Omega_\mathcal{N} \).
2. Given an ERL formula \( \varphi \), build a reduced ON \( \mathcal{N} \) such that \( \Omega_\mathcal{N} = [\varphi] \) (if such a reduced ON exists),

3. Given a reduced ON \( \mathcal{N} \), build a tight net \( \mathcal{N}' \) such that \( \Phi_\mathcal{N}' \equiv \Phi_\mathcal{N} \)
   (i.e. \( \Omega_\mathcal{N}' = \Omega_\mathcal{N} \)).
From Finite ONs to ERL Formulas

For a reduced ON $\mathcal{N}$, $\Phi_{\mathcal{N}}$ is built as follows:

**Building $\Phi_{\mathcal{N}}$**

\[
\Phi_{\mathcal{N}} = \bigwedge_{a, b \in \Psi, a < b} (b \rightarrow a) \land \bigwedge_{a, b \in \Psi, a \not\approx b} (\neg a \lor \neg b) \land \bigwedge_{a \in \Psi} \left(\left(\bigwedge_{b \in \Psi, b \leq a} b \rightarrow (a \lor \bigvee_{c \in \Psi, c \#_d a} c)\right)\land a \text{ enabled}\right)
\]

(causal closure) (conflict-freeness) (progress assumption)

where $\prec$ is defined as $a \prec b \overset{\text{def}}{=} (a < b) \land (\nexists c : a < c \land c < b)$.

progress assumption: “for any facet $a$, if $a$ is enabled, then $a$ or a direct conflict with $a$ has to fire”.

**Property**

$[\Phi_{\mathcal{N}}] = \Omega_{\mathcal{N}}$
From Finite ONs to ERL Formulas

Building $\Phi_N$

$$\Phi_N \equiv \bigwedge_{a,b \in \Psi, a \prec b} (b \rightarrow a)$$

(causal closure)

$$\bigwedge_{a,b \in \Psi, a \not\prec d b} (\neg a \lor \neg b)$$

(conflict-freeness)

$$\bigwedge_{a \in \Psi} ((\bigwedge_{b \in \Psi, b \prec a} b) \rightarrow (a \lor \bigvee_{c \in \Psi, c \not\prec d a} c))$$

(progress assumption)
From Finite ONs to ERL Formulas

Example

$$\Phi_N \equiv (c' \rightarrow b) \land (c \rightarrow b) \land (c \rightarrow a) \land (d \rightarrow a)$$
$$\land (\bar{a}' \lor \bar{a}) \land (\bar{b}' \lor \bar{b}) \land (\bar{c}' \lor \bar{c}) \land (\bar{c} \lor \bar{d})$$
$$\land \psi_{\bot} \land ((a \land b) \rightarrow (c \lor c' \lor d))$$
$$\land (a \rightarrow (c \lor d)) \land (b \rightarrow (c' \lor c))$$
$$\land (\psi_{\bot} \rightarrow (b' \lor b)) \land (\psi_{\bot} \rightarrow (a' \lor a))$$

(causal closure)
(conflict-freeness)
(progress assumption)
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**Procedure**

ϕ is an ERL formula on ψ⊥, a, a', b, b', c.

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Extracting the binary minimal constraints from ϕ

- Reminder: ▷; and #; define # (i.e. define Ω)
- \( a \triangleright b \iff \varphi \rightarrow (a \rightarrow b) \) is a tautology.
- \( a \not\equiv b \iff \varphi \rightarrow (\neg a \lor \neg b) \) is a tautology.
- This is decidable (co-NP complete).
**Procedure**

**Immediate conflicts**

\[ a \#_i b \rightarrow \psi \perp \]

\[ a \quad b \]

We call \( \text{CN}(\varphi) \) the net synthesized from \( \varphi \).
Procedure

Example

Binary minimal constraints

\[
\begin{align*}
& a \not\equiv_i a' \\
& b \not\equiv_i b' \\
& c \triangleright_i a \\
& c \triangleright_i b \\
& a \triangleright_i \psi_\bot \\
& a' \triangleright_i \psi_\bot \\
& b \triangleright_i \psi_\bot \\
& b' \triangleright_i \psi_\bot \\
\end{align*}
\]

\[\text{CN}(\varphi)\]
Lemma: Correctness of the construction

Let $\mathcal{N}$ be a finite reduced ON, then $CN(\Phi_{\mathcal{N}})$ is a tight net and $\Phi_{CN(\Phi_{\mathcal{N}})} \equiv \Phi_{\mathcal{N}}$.

Proof sketch.

1. $CN(\Phi_{\mathcal{N}})$ is an ON
2. $CN(\Phi_{\mathcal{N}})$ is reduced (because $\mathcal{N}$ is reduced)
3. $CN(\Phi_{\mathcal{N}})$ is tight (by construction)
4. $\Phi_{CN(\Phi_{\mathcal{N}})} \equiv \Phi_{\mathcal{N}}$ (same conflict relation)
Lemma: Correctness of the construction

Let $\mathcal{N}$ be a finite reduced ON, then $\text{CN}(\Phi_{\mathcal{N}})$ is a tight net and $\Phi_{\text{CN}(\Phi_{\mathcal{N}})} \equiv \Phi_{\mathcal{N}}$.

Proof sketch.

1. $\text{CN}(\Phi_{\mathcal{N}})$ is an ON
2. $\text{CN}(\Phi_{\mathcal{N}})$ is reduced (because $\mathcal{N}$ is reduced)
3. $\text{CN}(\Phi_{\mathcal{N}})$ is tight (by construction)
4. $\Phi_{\text{CN}(\Phi_{\mathcal{N}})} \equiv \Phi_{\mathcal{N}}$ (same conflict relation)

Theorem

Let $\varphi$ be an ERL formula, there exists a finite reduced ON $\mathcal{N}$ such that $\Phi_{\mathcal{N}} \equiv \varphi$ iff $\text{CN}(\varphi)$ is a reduced ON and $\Phi_{\text{CN}(\varphi)} \equiv \varphi$.

Proof.

$(\Rightarrow)$ If there exists a reduced ON $\mathcal{N}$ such that $\Phi_{\mathcal{N}} \equiv \varphi$, then, by the previous lemma, $\text{CN}(\varphi)$ is a candidate.

$(\Leftarrow)$ We choose $\mathcal{N} = \text{CN}(\varphi)$. 
Given \( \varphi \), \( \exists \mathcal{N} : \Omega_\mathcal{N} = [\varphi] \)

**Example 1**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Constraints</th>
</tr>
</thead>
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| \( \varphi = \psi_\bot \land (\bar{a} \lor \bar{b}) \) | \( \begin{align*}
  a & \not\equiv_i b \\
  a & \triangleright_i \psi_\bot \\
  b & \triangleright_i \psi_\bot
\end{align*} \) |

\[
[\varphi] = \{ \{ \psi_\bot \} , \{ \psi_\bot , a \} , \{ \psi_\bot , b \} \} .
\]

\( \text{CN}(\varphi) \) is a reduced ON but \( \Omega_{\text{CN}(\varphi)} = \{ \{ \psi_\bot , a \} , \{ \psi_\bot , b \} \} \neq [\varphi] . \)
Given $\varphi$, $\exists N : \Omega_N = \llbracket \varphi \rrbracket$

**Example 1**

**Formula**

$\varphi = \psi_\bot \land (\bar{a} \lor \bar{b}) \land (a \lor b)$

**Constraints**

- $a \not\equiv_i b$
- $a \triangleright_i \psi_\bot$
- $b \triangleright_i \psi_\bot$

$\llbracket \varphi \rrbracket = \{ \{\psi_\bot\}, \{\psi_\bot, a\}, \{\psi_\bot, b\} \}$. 

$\text{CN}(\varphi)$ is a reduced ON but $\Omega_{\text{CN}(\varphi)} = \{ \{\psi_\bot, a\}, \{\psi_\bot, b\} \} \neq \llbracket \varphi \rrbracket$. 
Given $\varphi$, $\exists N : \Omega_N = \llbracket \varphi \rrbracket$

Example 2

Formula

$\varphi = \psi_\bot \land (a \rightarrow c) \land (b' \rightarrow c) \land (b' \rightarrow a') \land (\bar{a} \lor \bar{a'}) \land (\bar{b} \lor \bar{b'}) \land (a \lor a') \land (b \lor b') \land (c \rightarrow (a \lor b'))$

Constraints

- $a \nmid_i a'$
- $b \nmid_i b'$
- $a \triangleright_i b$
- $a \triangleright_i c$
- $b' \triangleright_i a'$
- $b' \triangleright_i c$
- $b \triangleright_i \psi_\bot$
- $a' \triangleright_i \psi_\bot$
- $c \triangleright_i \psi_\bot$

$\llbracket \varphi \rrbracket = \{ \{ \psi_\bot, a, b, c \}, \{ \psi_\bot, a', b', c \}, \{ \psi_\bot, a', b \} \}$

The synthesized net is not an ON because there are two minimal events, $c$ and $\psi_\bot$. 
Given $\mathcal{N}$, build a tight net $\mathcal{N}'$ such that $\Omega_{\mathcal{N}'} = \Omega_{\mathcal{N}}$

Example 1

$$\Omega = \{\{\psi \perp, a, b, c\}, \{\psi \perp, a, b'\}, \{\psi \perp, a', b\}, \{\psi \perp, a', b'\}\}$$
Given $\mathcal{N}$, build a tight net $\mathcal{N}'$ such that $\Omega_{\mathcal{N}'} = \Omega_{\mathcal{N}}$

Example 2

$\Omega = \{\{\psi_\perp, a, b, c\}, \{\psi_\perp, a, b'\}, \{\psi_\perp, a', b\}\}$
Conclusion

Summary

- Occurrence nets with the maximal semantics,
- Concurrency is not logical independency,
- Generalization of the reveals relation into a framework for the description of logical dependencies between event occurrences in occurrence nets,
  - Definition of the ERL logic,
  - ON $\rightarrow$ ERL formula,
  - Synthesis: ERL formula $\rightarrow$ tight net (importance of the binary relations).
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Summary

- Occurrence nets with the maximal semantics,
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Discussion

- Construction applicable to the general semantics,
- Refinement of the construction,
- Synthesis where the reveals relation is not necessarily represented by the causality,
- Infinite ONs ?